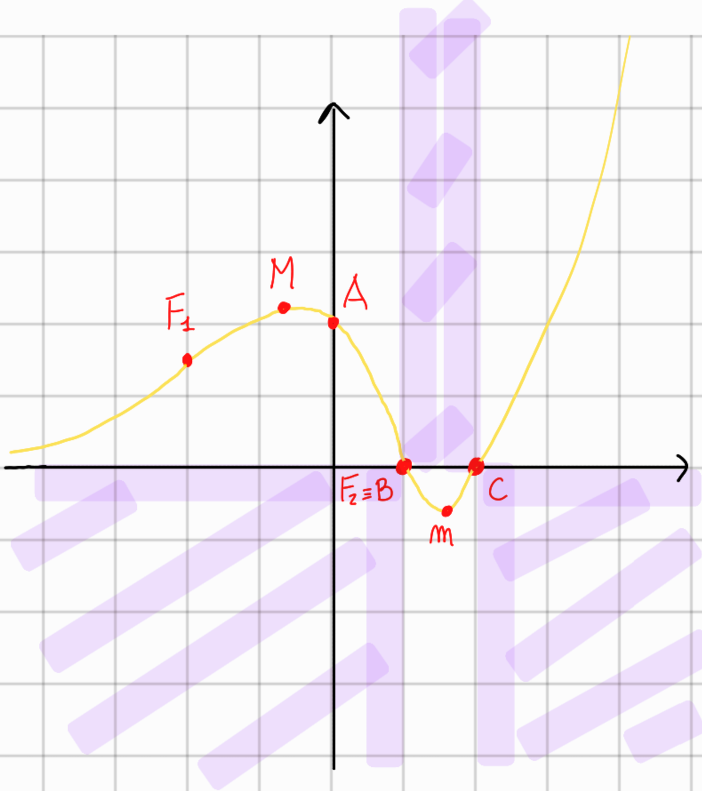
SOLUZIONI ESAME 29/01/24

ESERCIZIO 1

$$f(x) = e^x(x^2-3x+2)$$



DOMINIO : $\mathbb{R}$

INTERSEZIONE ASSE y : $f(0) = 1 \cdot 2 = 2$

$$A = (0,2)$$

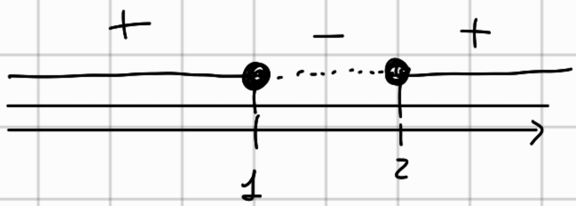
SEGNO E INTERSEZIONE ASSE X

$e^x(x-1)^2 \geq 0 \qquad e^x \geq 0 \quad \forall x \in \mathbb{R}$

$$x^2-3x+2 \geq 0 \quad \forall x \in \mathbb{R}$$

$$(x-1)(x-2) \geq 0$$

$x \leq 1 \quad \vee \; x \geq 2$



f NEGATIVA $\quad (1, 2)$

f POSITIVA $\quad (-\infty, 1) \cup (2, +\infty)$

f NULLA $\quad x = 1, \; x = 2$

$$B = (1,0) \quad , \quad C = (2,0)$$

LIMITI

$\lim_{x \to -\infty} e^x(x^2-3x+2) = 0$, $e^x \to 0$ ASINTOTO ORIZZONTALE SINISTRO

$\lim_{x \to +\infty} e^x(x^2-3x+2) = +\infty$ $\quad \left(\lim_{x \to +\infty} \frac{e^x(x^2-3x+2)}{x} = +\infty \quad \underline{\text{NO A. OBLIQUO DX}} \right)$

$\downarrow$

<u>NO</u> A.OR. DX

DERIVATA PRIMA $\qquad e^x(x^2-3x+2)$

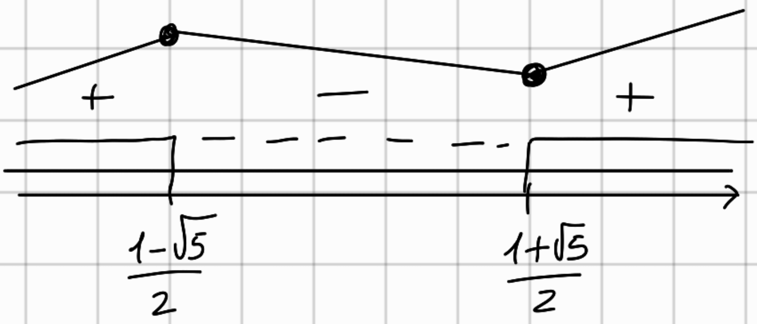
$$f'(x) = e^x(2x-3) + e^x(x^2-3x+2) = e^x(x^2-x-1)$$

$e^x \geq 0$ $\downarrow$ $\mathbb{R}$

$x^2-x-1 \geq 0$

$\Delta = 1+4 = 5$

$x_{12} = \dfrac{1 \pm \sqrt{5}}{2}$



f è DECRESCENTE $\left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

f è CRESCENTE $\left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$

f è STAZIONARIA $x = \dfrac{1 \pm \sqrt{5}}{2}$

$M = \left(\frac{1-\sqrt{5}}{2}, f\left(\frac{1-\sqrt{5}}{2}\right)\right) \simeq (-0.61, 2.28)$ MASSIMO

$m = \left(\frac{1+\sqrt{5}}{2}, f\left(\frac{1+\sqrt{5}}{2}\right)\right) \simeq (1.61, -1.19)$ MINIMO

DERIVATA SECONDA

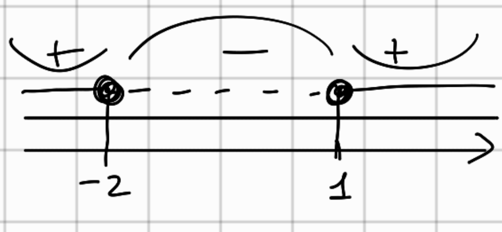
$$f''(x) = e^x(2x-1) + e^x(x^2-x-1) = e^x(x^2+x-2)$$

$e^x \geq 0 \ \forall x \in \mathbb{R}$

$x^2+x-2 \geq 0$

$\Delta = 1+8 = 9$

$x_{12} = \dfrac{-1 \pm 3}{2}$ $\nearrow x_1 = -2$ $\searrow x_2 = 1$



f CONVESSA $\quad x \in (-\infty, -2) \cup (1, +\infty)$

f CONCAVA $\quad x \in (-2, 1)$

f presente flesso $\quad x = -2, x = 1$

$F_1 = \left(-2, \frac{12}{e^2}\right) \simeq (-2, 1.62) \quad ; \quad F_2 = (1, 0) \equiv B$

(b) $f'(-2) = e^{-2}(4+2-1) = \dfrac{5}{e^2} > 0$

Poiché questo valore è positivo sappiamo che in quel punto la funzione è crescente e la retta tangente la funzione in $x_0 = -2$ avrà coefficiente angolare $\dfrac{5}{e^2} \simeq 0.67$

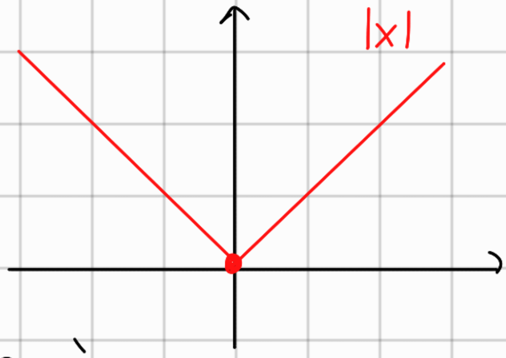
(c) Data $f: D \to \mathbb{R}$ un punto $x_0 \in D$ si dice P.TO ANGOLOSO SE

I LIMITI DX e SX DELLA DERIVATA PRIMA SONO DIVERSI E ALMENO UNO DEI DUE È FINITO $\ell, m \in \mathbb{R}$

$$\lim_{x \to x_0^-} f'(x) = \ell \neq \lim_{x \to x_0^+} f'(x) = m \ (\text{oppure } \infty)$$

OPPURE

$$\lim_{x \to x_0^+} f'(x) = \ell \neq \lim_{x \to x_0^-} f'(x) = m \ (\text{oppure } \infty)$$



0 È UN P.TO ANGOLOSO

$\boxed{\text{ESERCIZIO 2}}$

$$\int \frac{\ln x^2}{5x^2} = \frac{1}{5}\int x^{-2}(\ln x^2)\,dx = \frac{1}{5}\left[-\frac{1}{x}\ln x^2 - \int -\frac{1}{x}\cdot\frac{2}{x}\,dx\right] =$$

f' $\quad g$

$f = -\dfrac{1}{x}$ $\quad g' = \dfrac{2\cancel{x}}{x^{\cancel{2}}} = \dfrac{2}{x}$

$$= \frac{1}{5}\left[-\frac{1}{x}\ln x^2 + \int 2x^{-2}\,dx\right] = 5\left[-\frac{1}{x}\ln x^2 + 2\frac{x^{-1}}{-1}\right] =$$

$$= -\frac{1}{5}\left[\frac{\ln x^2 + 2}{x}\right]$$

$$\int_1^{e^2} \frac{\ln x^2}{5x^2}\,dx = -\frac{1}{5}\left[\frac{\ln e^4 + 2}{e^2} - \frac{\ln 1 + 2}{1}\right] = \frac{1}{5}\left[2 - \frac{6}{e^2}\right] \simeq 0.24$$

Questo valore rappresenta la misura dell'area sotto la curva $\dfrac{\ln x^2}{5x^2}$ tra $x = 1$ e $x = e^2$

ESERCIZIO 3

$$\int_0^1 3\pi t \sin(\pi t^2)\,dt =$$

$$-\frac{3}{2}\int_0^1 (-2\pi t)\sin(\pi t^2) = -\frac{3}{2}\cos\left[\pi t^2\right]_0^1 = -\frac{3}{2}\left[\cos(\pi)-\cos(0)\right]$$

$$-\frac{3}{2}\left[-1-1\right] = 3$$

ESERCIZIO 4 $h=3$ $k=3$ $(h-1)(k-1)=4$

-

22	11	17	50
13	37	30	80
25	12	33	70
60	60	80	200

15	15	20	50
24	24	32	80
21	21	28	70
60	60	80	200

$$\chi^2_4 = \frac{(22-15)^2}{15}+\frac{(11-15)^2}{15}+\frac{(17-20)^2}{20}+\frac{(13-24)^2}{24}+\frac{(37-24)^2}{24}+\frac{(30-32)^2}{32}+\frac{(25-21)^2}{21}+\frac{(12-21)^2}{21}+\frac{(33-28)^2}{28}=$$

$$=\underbrace{\frac{49}{15}+\frac{16}{15}}+\frac{9}{20}+\underbrace{\frac{121}{24}+\frac{169}{24}}+\frac{4}{32}+\underbrace{\frac{16}{21}+\frac{81}{21}}+\frac{25}{28}=$$

$$= 4.33+0.45+12.08+0.12+4.62+0.89 = 22.49 > 18.467$$

"fascia d'età" è una variabile qualitativa ORDINATA

"programma televisivo" è una variabile qualitativa NON ORDINATA

- $\bar{X} = \frac{1}{6}\left(6.0+5.8+4.0+6.1+8.9+5.2\right) = \frac{36}{6} = 6$

$$VAR(x) = \frac{1}{6}\left(0+0.2^2+2^2+0.1^2+2.9^2+0.8^2\right) = \frac{1}{6}\left(0.04+4+0.01+8.41+0.64\right) = 2.18$$

$$MEDIANA(x) = \frac{5.8+6.0}{2} = 5.9$$